Session Guide: Counting Squares

Session Outline

Description

In this session, learners are asked to count how many squares they can find in a given square grid. It is expected that learners will see this as a simple task of counting up the squares. Usually, learners tend to only count the unit squares before realising there are many more. When they start counting, they will likely find it hard to keep track of which squares they've already counted and which they haven't. In this puzzle, we are much more interested in the methodological approach to the counting than in the correct solution. It is then expected that they are encouraged to break down the large square into smaller units and record the respective counts systematically in order to find a method to count the number of squares in any square grid.

Session Objectives

The objectives of this session are to:

- Develop students' problem solving skills.
- Recognise patterns and apply them in different contexts.
- Be able to justify solutions to problems accurately.
- Learn how to record data and work systematically and in an organised manner.
- Gain confidence in sharing ideas openly.

Expected Outcomes

By the end of the session students will have:

- Considered a challenging problem by studying smaller similar problems.
- Tried to find a pattern that would help solve a challenging problem.
- Come up with hypotheses and test them.
- Applied methods developed to more complex situations.
- Produced evidence of their thinking process and working towards a general solution.

Areas involved

- Mathematics and financial literacy

    - Math brain teasers
    - Riddles
- Life skills
    - Organizational skills
    - Communication

Activity: Starter

Objectives

To engage students with a mathematical puzzle that is challenging to solve and allow them to identify the need to develop a strategy to solve it.

Expected Outcomes

- Students discuss potential approaches and strategies for solving the problem.
- Students propose answers or estimate what they think the correct answer is.

Teaching Instructions

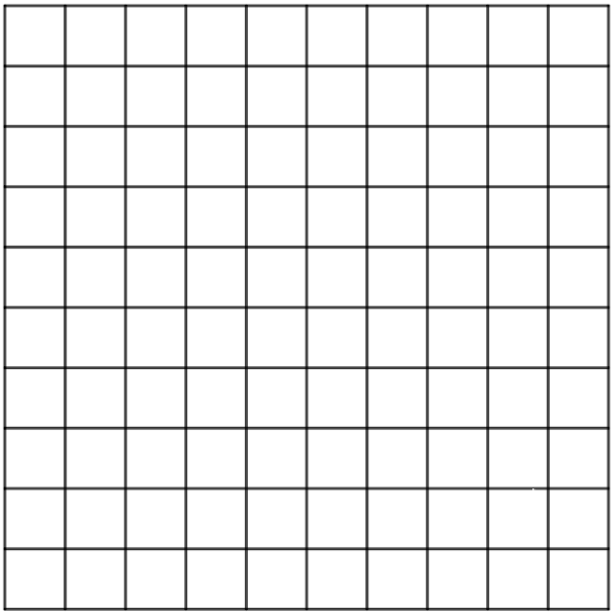
- Pose the problem and provide a short time for students to try it out.
- Encourage students to share answers.
- Allow students to suggest answers at any stage they want and give positive feedback, without stating if they are correct or not (“Have you considered all possibilities? Are you sure you counted every single possible square?”).
- Encourage students to explain how they found their answer.
- Do not provide the answer or solution to students, as this will come later in the session.

Suggested Guidelines

- Recommended time: 5-10 minutes.
- Students could work in pairs to encourage discussion and collaborative thinking (maintaining social distance!).
- Encourage incorrect answers, students will learn more from mistakes than from getting to a correct answer directly. For small answers respond with something like “I think you can find a few more squares”.
- As soon as students start to recognise how difficult this question actually is it is recommended to move the discussion towards the next activity.
- Encourage participation regardless of gender, ability, cultural background, age and where they live.

Student Instructions

If it is possible to print or project: How many squares can you see in this figure? (answer: 385)



If it's not possible the activity could be described as follows: If you draw a 10x10 grid split into 100 unit squares, how many squares are you drawing in total?

Activity: Introduction

Objectives

- To understand that large problems can sometimes be solved by considering smaller examples.
- To develop organised ways of working.
- To make inferences and find relations between examples.

Expected outcomes

Students understand how this problem relates to smaller similar problems that are easier to tackle. Students correctly identify the number of squares in the 1x1 case, the 2x2 case, the 3x3 case and the 4x4 case.

Teaching Instructions

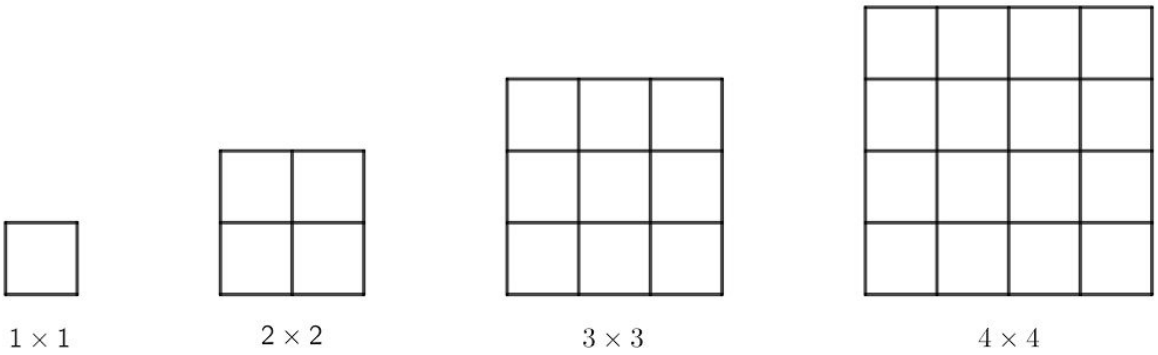
- Lead a discussion on how the different counts came up and what difficulties they experienced.
- Draw the conclusion that it is quite difficult to find the answer to the 10x10 problem. If there were several answers proposed in the starter, emphasise this to suggest that there is no certainty yet that any of the answers is correct.
- Explain that it is possible to consider similar problems of smaller size which might be easier to answer, and pose the problems.

Suggested Guidelines

- Recommended time: 10-15 minutes
- Suggested steps:
    - Draw out the conclusion that it is a hard problem.
    - If there are several suggested answers emphasise that this is the case and we need a strategy to solve this problem and verify if any of them is the correct answer.
    - Describe the strategy of looking at smaller examples.
    - Start with the 1x1 case and pose it as a question.
    - Go on to the 2x2 case and collect student answers and allow students to explain their thinking and processes.
    - Go on to the 3x3 and then the 4x4 case in a similar way.
- You can use a *Think, Pair, Share* technique to give students thinking time to individually consider the problem, allow them to check and discuss their ideas with a peer and then share their answers, either by discussing with individual groups or facilitating sharing to the entire group.
- Emphasise that a result alone is not useful, you also need an explanation on how you got to the result.

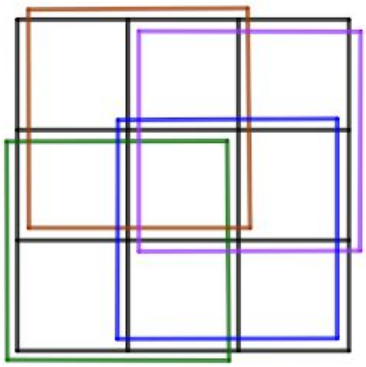
Student Instructions

If we now have a 1x1 grid, a 2x2 grid, a 3x3 grid and a 4x4 grid, can we count the number of squares in each case? (answers: 1x1: 1, 2x2: 5, 3x3: 14, 4x4: 30)



Note

It can be difficult to count the number of 2x2 squares in a 3x3 grid so it might be useful after giving some time to show how this can be achieved nicely as described in the following image:



You can also find a video that demonstrates the counting process for a 5x5 grid here.

Activity: Towards a Pattern

Objectives

- To make inferences and find relations between examples.
- To work systematically.
- To collect and record data in an organised manner.
- To recognise patterns.

Expected Outcomes

Students should be able to describe their solution by breaking it up into categories. Students should understand the relation between one case and the next and use it to find a pattern in the answers.

Teacher Instructions

- (Activity 1) Ask students if they can find a pattern or relationship between their answers for the 1x1 grid, the 2x2 grid, the 3x3 grid and the 4x4 grid.
    - If they find a correct pattern, ask them to justify it.
    - If they didn't find a pattern, give them some time to think about it, emphasising the importance of justifying why it works.
- (Activity 2) Explain that in order to find and justify a pattern it can be useful to categorise the types of answers they are getting for each square. Help students define the categories by distinguishing the size of the different squares in each grid and counting them separately. Suggest organising this in a table format.
- Give students time to fill in the table and try to spot a pattern.
- (Activity 3) Can we now find a pattern and justify why it occurs?

Suggested Guidelines

- Suggested time: 15 minutes.
- Encourage students to think about the relation between one case and the next, emphasising that there is one. You can ask questions like "How do you think the first case relates to the second? How does the second relate to the third? How about the third and the fourth?". If they are struggling (after some time) ask how many times the 1x1 square fits in the 2x2, how many times the 2x2 fits in the 3x3, how many times the 3x3 fits in the 4x4.
- Ask students to try to find it and help them discover it if they are struggling. Make sure students have enough time to think and allow them to share ideas emphasising that all ideas are valued.
- Once they have the numbers in the table, if they still cannot find a pattern encourage them to think along diagonals.
- To test they understand the pattern you can ask them to extend the table to the 5x5 grid.
- Draw attention to the fact that the numbers in the table are square numbers. Why is this?

Student Instructions

Activity 1

We now have the numbers of squares in a 1x1 grid, in a 2x2 grid, in a 3x3 grid and a 4x4 grid. Is there any pattern in your results? Can you explain why this pattern occurs?

Activity 2

Can we write our answers in an organised way, for example by splitting the answers into the sizes of squares we can find? Perhaps we can use a table (filled in for teachers):

Problem size	1x1	2x2	3x3	4x4	Total
1x1	1				1
2x2	4	1			1+4=5
3x3	9	4	1		1+4+9=14
4x4	16	9	4	1	1+4+9+16=30

Activity 3

Can we find a pattern in our answer?

Note

The pattern is as follows:

- For the 1x1 we have 1 1x1 square.
- For the 2x2 we have 1 2x2 square and 4 1x1 squares, so 1 + 4 = 5 in total.
- For the 3x3 we have 1 3x3 square, 4 2x2 squares and 9 1x1 squares, so 1 + 4 + 9 = 14 in total.
- For the 4x4 we have 1 4x4 square, 4 3x3 squares, 9 2x2 squares and 16 1x1 squares , so 1 + 4 + 9 +16 = 30 in total.

Activity: Plenary

Objective

To be able to arrive at general conclusions, justify them and use them in other contexts.

Expected Outcomes

Students should be able to explain their conclusion for their pattern, use it to find the answer to the 10x10 case and explain how they could find the answer for a 20x20 grid, a 100x100 grid or a grid of any size.

Teacher Instructions

- Make sure students understand the pattern. You can ask new students to explain the pattern again if needed and why it works.
- Ask students how many squares are there in the 10x10 grid.
- Ask students how many squares there should be in a 20x20 or a 100x100 grid.

Suggested Guidelines

- The explanation is important but if the students are finding the activity too difficult, then just finding the pattern is good enough. More able students could try to write a justification as a challenge, either during the session or as homework.
- Encourage the answer to be given as $1^2+2^2+3^2+4^2+...+10^2$ as this demonstrates better understanding than the number 385. The same applies to the larger cases, we are not interested in students being able to carry out the arithmetic, instead we are interested in their understanding of how to get to the correct numerical answer.

Student Instructions

How many squares are there in the initial 10x10 square? Why? Can you tell me how you could find out how many squares there would be in a 20x20 square? How about a 100x100?