Session Guide: 4 Colour Theorem

Session Outline

Description

In this session learners explore the 4 colour theorem, a famous theorem known for being the first theorem proved using computers. Learners will start by exploring the idea of a map and a colouring. They will then explore when two maps are the same and how different colourings can be achieved with the minimal number of colours. Finally they will be posed the challenge of finding a counterexample for the theorem, i.e. creating a map that requires 5 or more colours, in order to conclude that all maps can be coloured using 4 colours.

Session Objectives

The objectives of this session are to:

- Explore an accessible but complex mathematical concept.
- Develop abstraction skills and strategic thinking.
- Make inferences and test them.

Expected Outcomes

By the end of the session learners will have:

- Understood the concepts involved in the 4 colour theorem.
- Developed strategies to colour maps using the minimum number of colours possible.
- Created maps for themselves and their peers to explore.
- Worked with complex maps.

Areas involved

- Mathematics and Financial literacy
    - Geometry shapes
    - Math Brain Teasers
- Creative Arts
    - Draw and paint pictures
- Life skills

- Study and organisational skills

Activity: Understanding Map Colourings

Objectives

To understand the main idea about colouring maps.
To be able to colour a map accurately using few colours.

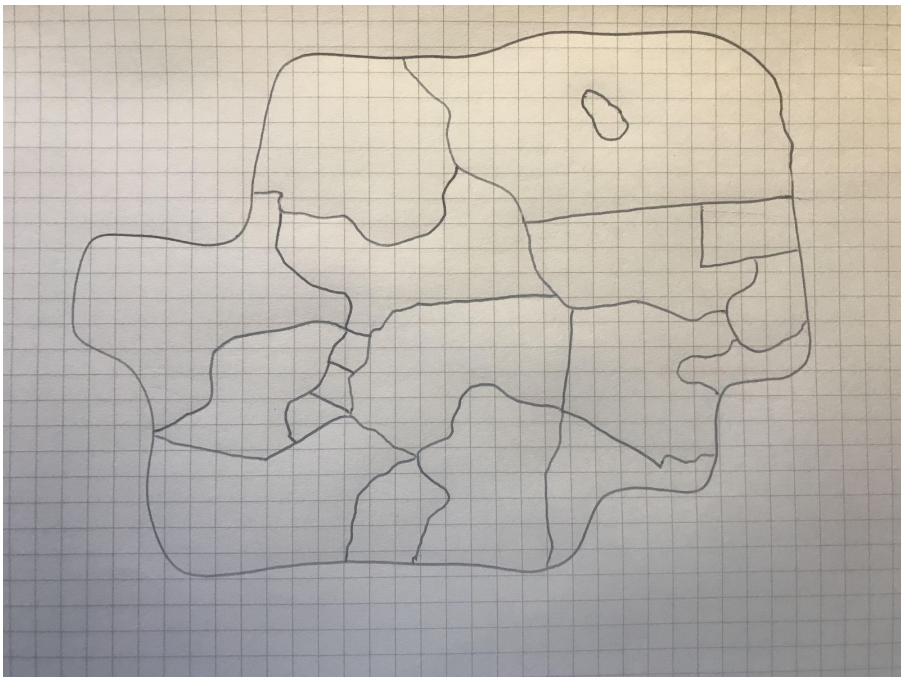
Expected Outcomes

Learners will have created some maps and coloured them with as few colours as possible.

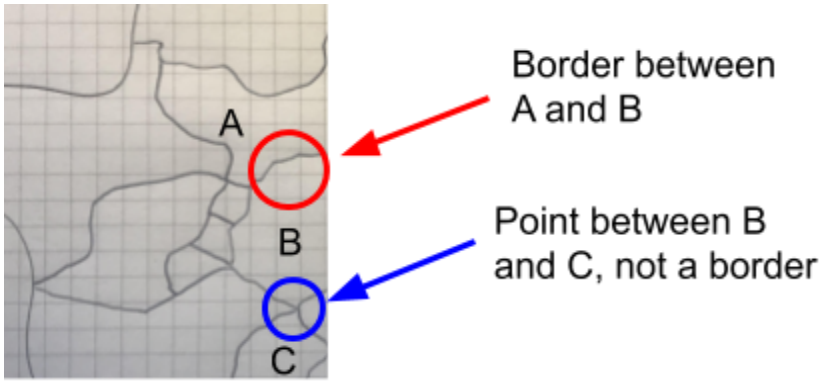
Teaching Instructions

Learners should draw a fictional map with at least 15 regions in it. They should then colour their map using as few colours as possible so that regions that have a border with each other cannot have the same colour. Note that a border is there only if there is a line between two regions, a common point is not considered a border. If colours are not available they could represent colours by numbers and fill in each region with numbers instead. Learners should then share how many colours they managed to use to colour their map.
Below are some examples that demonstrate the key points.

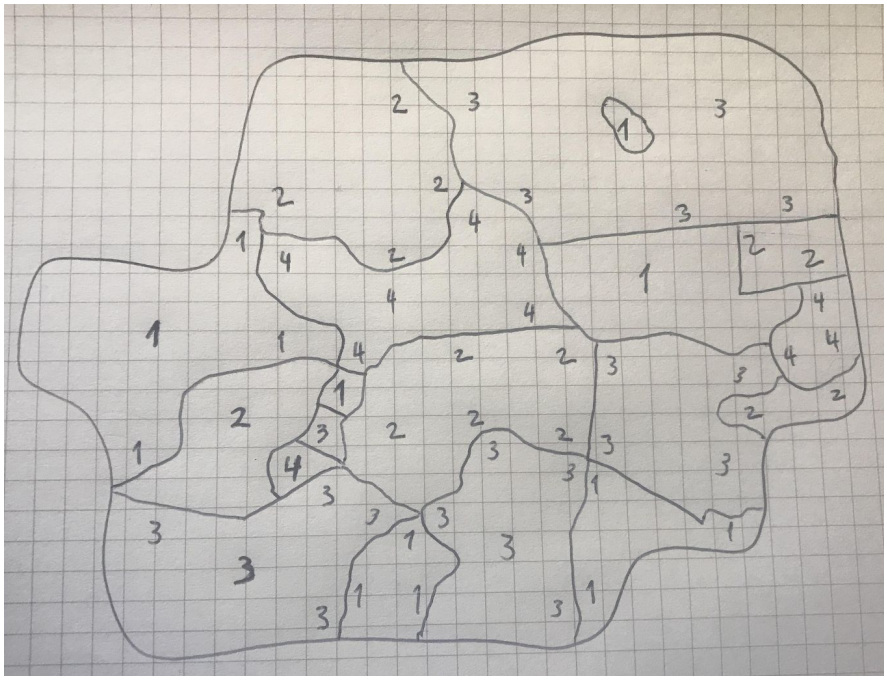
- Example of a possible map. It contains both straight and not-straight borders between regions.



- Distinction between a border and a common point. The red circle shows a border while the blue shows a common point which is not considered a border.



- Map coloured using numbers. Numbers can be drawn several times within the same region to make sure that different colours between neighbouring regions are clearly visible.



Student Instructions

Draw a fictional map divided into at least 15 regions. Colour your map with a colour for each region using as few colours as possible so that regions that have a border with each other cannot have the same colour. Note that a border is there only if there is a line between two regions, a common point is not considered a border. If colours are not available you could represent colours by numbers and fill in each region with numbers instead. What is the smallest number of colours you managed to use?

Activity: Colouring the Same Map

Objectives

To determine that there are many different colourings for a map using the same number of colours.
To work systematically to minimise the number of colours used.
To understand what is meant by the same map.

Expected Outcomes

Learners will have worked on a map to colour in different ways to minimise the number of colours used.
Learners will have compared their colourings with their peers' colourings.

Teaching Instructions

Split learners into groups of 2 or 3. Learners should draw a new map of 8 to 12 regions trying to make it difficult to colour using a small number of colours. They should then show their maps to the other members of their team and select one that they think is particularly challenging. Discuss with the whole group what is meant for two maps to be the same (they can look slightly different but as long as all the borders between regions are respected the maps are essentially the same). Members of the teams should make 3 copies of the chosen map. They should then independently colour their first map and compare it with the rest of the group, discussing strategies to minimise the number of colours used. They should then repeat this with the second copy of the map, trying to reduce the number of colours, discuss their outcomes within the group and repeat with the final map.
Lead a discussion on their investigation, focused on whether they managed to use a particular strategy to minimise the number of colours, whether they managed to reduce the number of colours in each attempt and what the smallest number of colours they needed in each case.

Suggested Guidelines

- When making copies of a map it might help to give a name to each region, perhaps using a letter for each one.

Student Instructions

Draw a new map that you think is going to be difficult to colour with a small number of colours. Make 2 additional copies of this map, making sure that even if they don't look exactly the same, the borders between regions are the same and any two regions that have a border, they share this border in all copies. Colour the first one and think about how you could have coloured it differently to reduce the number of colours needed. Repeat with the third map. How many colours did you manage to use? Was this number the same in all three attempts?

Activity: A 4-Colour Map

Objectives

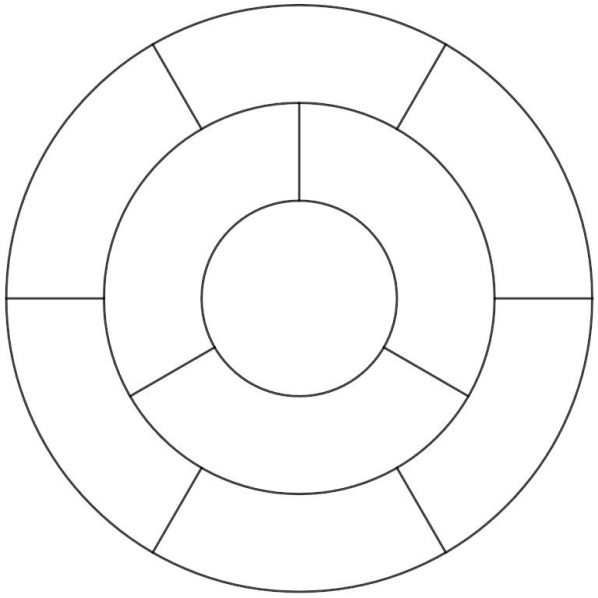
To develop further strategies and requirements to minimise the number of colours in a map.
To make conjectures about the minimal number of colours to colour any map.

Expected Outcomes

Learners will have attempted to minimise the number of colours used for a specific, complicated map.
Learners will have made conjectures and tested them.

Teaching Instructions

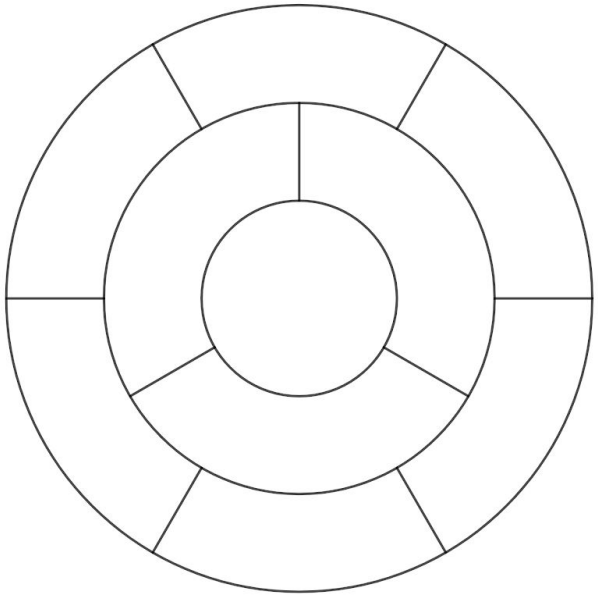
Present the following map to learners to copy and colour. Learners should then share the number of colours they managed and discuss how they might have been able to reduce the number of colours used if possible (this map requires 4 colours).



Afterwards pose the challenge of creating a map that requires 5 or more colours. If a learner thinks they managed, they should present the map to the rest to check whether they can colour it with fewer colours.
Finally ask learners what is the largest number of colours required to colour any map.

Student Instructions

Colour the following map with as few colour as possible:



How many colours did you use? Can you use fewer colours? Can you come up with a map that requires 5 colours to colour it?