Session Guide: Sum of Consecutive Integers

Session Outline

Description

This session explores a number of aspects related to writing numbers as the sum of consecutive integers. This is a problem that has engaged mathematicians throughout the history of modern mathematics at least. It presents learners with three activities on this theme, focusing on justification of results, and proposes three questions for further investigation. A key aspect of the session is determining whether justifications are correct and complete. Solutions for the more complicated activities are presented in the description of the activity for reference.

Session Objectives

The objectives of this session are to:

- Develop abstraction skills.
- Explore simple mathematical concepts that can lead to sophisticated conclusions.
- Work imaginatively with numbers.

Expected Outcomes

By the end of the session learners will have:

- Explored a numerical problem in detail.
- Justified arguments accurately.
- Decided when a justification is complete.

Areas involved

- Mathematics and Financial literacy
  - Math Brain Teasers
  - The Number System
  - Expressions and Equations
- Life skills
  - Study and organisational skills

- Making decisions

Activity: Starter Challenge

Objectives

To gain familiarity with the concept of the problem for this session.

Expected Outcomes

Learners will have found as many integers as possible that can be written as the sum of consecutive numbers in 5 minutes.

Teaching Instructions

Pose the challenge of finding as many integers (whole numbers) between 1 and 20 that can be written as a sum of positive consecutive numbers, for example, 5 can be written as 2+3, giving them 5 minutes to do so. After the 5 minutes, go through numbers from 1 to 20 asking if any learner managed that number and how. For each number ask if it can be done in any different way (several numbers can be written as a sum of consecutive numbers in different ways, for example 15 = 7+8 = 4+5+6 = 1+2+3+4+5).

Suggested Guidelines

- It is likely that most learners will only try to write numbers as the sum of only two consecutive numbers. If you notice that this is the case for most learners you can give them the hint that more than two consecutive numbers can be used.

Student Instructions

Some integers (whole numbers) can be written as a sum of consecutive positive integers. For example, 5 = 2+3. How many numbers between 1 and 20 can you write as the sum of consecutive integers in 5 minutes?

Activity: Multiples of 3

Objectives

To be able to make conclusions and justify them.

Expected Outcomes

Learners will have understood why all multiples of 3 can be written as the sum of three consecutive positive integers.

Teaching Instructions

Learners should focus on multiples of 3. Can all multiples of 3 can be written as the sum of consecutive positive integers? If so, why? Present the question to learners, give them time to think about it individually, then discuss it with one or two peers and then share with the whole group. The group should come up with a final answer with a clear explanation.

Answer (do not share!): All multiples of 3 can be written as the sum of three consecutive integers. In the case of 3 it can be done as 1+2. For all other multiples of 3 you can simply divide it by 3 and consider the successor and predecessor, for example if you consider 12 and divide it by 3 you get 4, which has a predecessor of 3 and a successor of 5, and 3+4+5=12. Algebraically, a multiple of 3 can be written as $3n$. Dividing it by 3 you get n, which has a predecessor of $n-1$ and a successor of $n+1$. Adding them you get $(n-1)+n+(n+1) = n+n+n = 3n$, which is the original multiple of 5.

Suggested Guidelines

- The important point here is the justification, ideally algebraically but otherwise in words. If there is no complete justification then the problem is not solved.

Student Instructions

Can all multiples of 3 be written as a sum of consecutive positive integers? If so, why? If not, can you find an example of a multiple of 3 that cannot be written in such a way?

Activity: Multiples of 5

Objectives

To be able to make conclusions and justify them.

Expected Outcomes

Learners will have understood why all multiples of 5 can be written as the sum of three consecutive positive integers.

Teaching Instructions

- Repeat the previous activity with multiples of 5.

  Answer (do not share!): This is almost exactly the same as the case of multiples of 3 but with two predecessors and two successors. Formally, we need to consider 5 and 10 first, which can be written as 5 = 2+3 and 10=1+2+3+4. For any other multiple of 5, it can be written as $5n$ and dividing it by 5 we get n. The two predecessors are $n-2$ and $n-1$ and the two successors are $n+1$ and $n+2$. Adding them gives $(n-2)+(n-1)+n+(n+1)+(n+2) = 5n$, the original multiple of 5. Note that we have to consider the case of 5 and 10 separately as if we apply the process to these numbers, the second predecessors are -1 and 0 respectively, which are not positive, so we have to consider them separately.

Student Instructions

Can all multiples of 5 be written as a sum of consecutive positive integers? If so, why? If not, can you find an example of a multiple of 5 that cannot be written in such a way?

Activity: Open Questions

Objectives

To investigate a number of questions related to the problem.

Expected Outcomes

Learners will have explored a number of related questions and shared proposed answers with justifications.

Teaching Instructions

Pose the following questions and allow learners to discuss them in small groups. If any group thinks they have found an answer to any of the questions they should share them with the whole group and justify it. The group should question any potential faults in the arguments until they are all convinced they are correct.

1. Can the argument for multiples of 3 and 5 be expanded to multiples of any odd number?
2. Can all integers greater than 2 be written as the sum of consecutive positive integers? If not, which ones cannot be done?
3. Some numbers can be written as the sum of consecutive positive integers in more than one way. Which numbers can only be written as such a sum in only one way? Which ones in 2 ways? Which ones in more than 2 ways?

Suggested Guidelines

- If you are not convinced about the answers from your learners, you can ask them to write their solution down and email a picture to contact@somanyumbani.com and we will try to get back to you with comments.

Student Instructions

Consider the following questions, investigate them separately and try to come up with an answer and s justification for your answer.

1. Can the argument for multiples of 3 and 5 be expanded to multiples of any odd number?
2. Can all integers greater than 2 be written as the sum of consecutive positive integers? If not, which ones cannot be done?
3. Some numbers can be written as the sum of consecutive positive integers in more than one way. Which numbers can only be written as such a sum in only one way? Which ones in 2 ways? Which ones in more than 2 ways?