Session Guide: Calendar Squares

Session Outline

Description

This session introduces a simple but interesting mathematical investigation about ways of organising numbers in a table, starting from a simple organisation they are familiar with: calendars. The problem is initially presented through a very simple investigation with an accessible justification for the final result, and step-by-step it presents complexities and generalisations for learners to engage with. It concludes with a number of open questions that learners can investigate independently.

Session Objectives

The objectives of this session are to:

- Improve problem solving skills.
- Develop problem posing skills.
- Recognise patterns and regularities.
- Make inferences, test them and justify them.
- Work collaboratively.
- Assess the completeness of mathematical arguments or proofs.

Expected Outcomes

By the end of the session learners will have:

- Explored a simple problem involving squares in calendars.
- Generalised the problem and tested their results also hold in more complex cases.
- Considered further versions of the problem and created their own related problems.

Areas involved

- Mathematics and Financial Literacy
    - Math Brain Teasers
    - The Number System
    - Expressions and Equations
- Life skills

- Making decisions
- Communication
- Study and organisational skills

Activity: Diagonals in 2x2 Squares

Objectives

- To understand the basic concept of the problems.
- To make inferences, test them and justify them.

Expected Outcomes

- Learners will have explored the problem in its most basic form, the sum of diagonals in a 2x2 square.
- Learners will have reached conclusions and tried to justify them.

Teaching Instructions

Present the initial problem: Write a calendar month in its weekly format as a table with 7 columns and as many rows as needed. The month can start on any day of the week and have 28 to 31 days. It is the learner's choice. For example, a month could look as follows:

		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

Ask learners to select a 2x2 square that includes for days in the month and add the numbers in the diagonals. For example, we can select the following square:

		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26

27	28	29	30			

The sum of the diagonals would be 8 + 16 = 24 and 9 + 15 = 24.
Ask learners if they notice anything. Ask them to repeat for other squares within the month, or even try designing another month and try it out. Why does this work? Lead a discussion and let the group decide when the justification is complete.

Answer (do not share!): if we call the top-left number x, then the top-right number is the day after, so it's x+1, the bottom-left number is one week after the first one so it is x+7, and the bottom-right is the day after the bottom-left so it is x+8. If we add the diagonals we get top-right + bottom-left = x+(x+8) = 2x+8, and top-left + bottom-right = (x+1)+(x+7) = 2x+8 and so both diagonal sums are always the same.

Suggested Guidelines

- This will work for any design of month (different number of days or start day for the month. Encourage learners to use different designs to emphasise this point.

Student Instructions

Imagine you have a calendar for a month. This is a table with 7 columns, one for each day in the week, and 4 to 6 rows, with numbers from 1 to the end of the month depending on what month it is (28, 29, 30 or 31). Draw a sample calendar month for any month, choosing how long the month should be and the first day on the month.

Select a 2x2 square in your calendar selecting 4 numbers. Add the numbers in each diagonal. What do you get for the sum of each diagonal? What do you notice? Is this the same for all 2x2 squares or for any other months that you could design (different length of the month and different starting weekday)? If so, why? Can you justify your answers?

Activity: Larger Squares

Objectives

- To be able to expand and understand more complex versions of the problem.
- To make inferences, test them and justify them.

Expected Outcomes

- Learners will have explored larger squares in the month, considering as well middle rows and middle columns for the 3x3 and 5x5 (where possible) cases.
- Learners will have reached conclusions and tried to justify them.

Teaching Instructions

Expand the problem to consider larger squares. Does the same result hold? If so why? In the case of 3x3 squares, what happens when you also add the numbers in the middle row and in the middle column? Do we get any different results? If you design a calendar that includes possible 5x5 squares, does the same apply, including the cases of the middle row and column?

As above, pose the problem and let learners investigate it, share their results and conclusions, ask for justifications and let the group decide when the justification is complete. The results follow a very similar argument as the one above.

Student Instructions

Draw a few new calendars for different months. Draw larger squares of size 3x3, 4x4 and if possible 5x5. Add the diagonals in the squares. Do you notice anything? In the cases 3x3 and 5x5, add the numbers in the middle row and in the middle column. Does the result still follow? Write a short paragraph explaining why anything you notice happens.

Activity: Generalising The Results

Objectives

- To explore new problems that arise from a simple problem.
- To learn how to pose questions.
- To make inferences, test them and justify them.

Expected Outcomes

- Learners will have explored a more general case of this problem.
- Learners will have understood how solutions for one problem can be adapted to other problems.

Teaching Instructions

Now instead of looking at calendars, extend the problem to a table of consecutive numbers where the first number could be anything the learners choose. For example, one such table could be:

8	9	10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29	30	31
32	33	34	35	36	37	38	39	40	41	42	43
44	45	46	47	48	49	50	51	52	53	54	55
56	57	58	59	60	61	62	63	64	65	66	67
68	69	70	71	72	73	74	75	76	77	78	79

Split the group into teams of 3 or 4. Each team should come up with a few tables of different sizes and starting values. They should then draw squares of different sizes and add the diagonals, and in the cases of 3x3, 5x5, 7x7, … squares they should also add the middle rows and columns in their squares. Do they notice anything? Why does this happen? Teams should write a paragraph explaining why this happens. Instead of describing what happens on individual tables they should try to write it for any table, using r for the number of rows, c for the number of columns x for a number within their square and n for the size of their square (nxn).
Each team then should share their results with the whole group. Different teams should comment on the solutions presented, trying to identify at least one good point and if possible one point for improvement.

Suggested Guidelines

- If learners are struggling with Algebra you can allow them to describe it in words rather than algebraically.

Student Instructions

Now instead of looking at a calendar-style table of numbers, write a table of any size you want (ideally not with 7 columns as before!), a starting number of your choice at the top-right corner and consecutive numbers in a similar way as the calendar.

Draw several squares of different sizes and add the diagonals. Do you notice anything? Repeat the process with other tables and other square sizes. Does the same happen? Write a paragraph to explain why this might happen. You may want to use c for the number of columns, r for the number of rows, x for the top-left number in your square and n for the size of your square (which has size nxn).

Activity: Open Question

Objectives

- To further generalise the problem.
- To develop problem posing skills.

Expected Outcomes

- Learners will have considered a further generalisation of the problem.
- Learners will have taken a further problem to consider independently.

Teaching Instructions

Ask learners what other problems they could consider involving these tables of numbers. Give them time to think about them and play around trying to find interesting patterns.

Pose the final open question: if instead of looking at squares within the table you look at rectangles and instead of adding diagonals you add opposite corners of the rectangle, what happens? Why might this be? Can you find any other interesting similar results, for example if you have a specific type of numbers in your table such as even numbers?

Suggested Guidelines

- Encourage learners to think about this independently and try to create an opportunity to come back to this and review any findings from the learners in a few days.

Student Instructions

If instead of looking at squares within the table you look at rectangles and instead of adding diagonals you add opposite corners of the rectangle, what happens? Why might this be? Can you find any other interesting similar results, for example if you have a specific type of numbers in your table such as even numbers?